1 Introduction

Problematic of warps

In the conventional understanding of galaxies embedded in a fat spheroidal or triaxial halos of dark matter, the frequently observed pronounced outer warps of the neutral hydrogen and, to a less noticable extend, optical disks set a challenge for explaining their presumably long lived existence, and particularly their frequent quasi-straight line of nodes. At least half the spirals possess a detectable warped HI and stellar disk, and more should remain undetected due to projection confusion.

Self-gravitating massive disk

Hypotheses like magnetic field (Battaner et al. 1990), discrete modes (Sparke 1984 and Sparke & Casertano 1988) misaligned dark halos (Dubinski & Kuijen 1995), or gravitational interactions (Hernquist 1991 [3]) have been proposed, without clearly matching all the pieces of the puzzle.

But, as argued by Reshetnikov & Combes 1998 and Arnaboldi et al. 1997 and Becquaert & Combes 1997, the evidences for non-self-gravitating HI disks are actually weak. Instead the opposite assumption of coexistence of warps with most of the dynamical mass is not in contradiction with the available observations. Such an assumption of thick and flaring disks of dark matter in the form of cold molecular gas related to neutral hydrogen has been proposed by Pfenniger et al. (1994) to account for many more known facts about spirals.

The massive disk assumption is attractive on the dynamical point of view because the straight line of node would automatically follow from the weak diffusive properties of angular momentum in a self-gravitating disk : if, following for example some accretion event, some angular momentum is deposited in a self-gravitating disk, bending its outer parts, the angular momentum will *slowly* diffuse across the whole disk in a monotonous way because angular momentum is a quasi conserved quantity. Because its diffusion is slow, the line of nodes must be quasi-straight and persist for many rotational periods.

Search for periodic orbits

In order to understand in a detailed way the dynamics of warped self-gravitating disks, it is useful to describe the backbone of galactic dynamics, the periodic orbits. Therefore in this paper we present succintely a study of periodic orbits in a simple model of warped disk galaxies. Different orbit families are found. The stability and consistence with the density distribution have been determined. The influence of a global rotation of the warp pattern is examined too.

2 Model

2.1 Warped potential

The galaxy potential is modeled by a triple Miyamoto-Nagai potential (bulb, dust disc and gas disc) containing a warping term. This term, depending on the cosine of azimuthal angle θ, induces an z-elevation of the outer disc above the plane tangent to the central disk. The z-height (Δz) depends on the square of the cylindrical radius R.

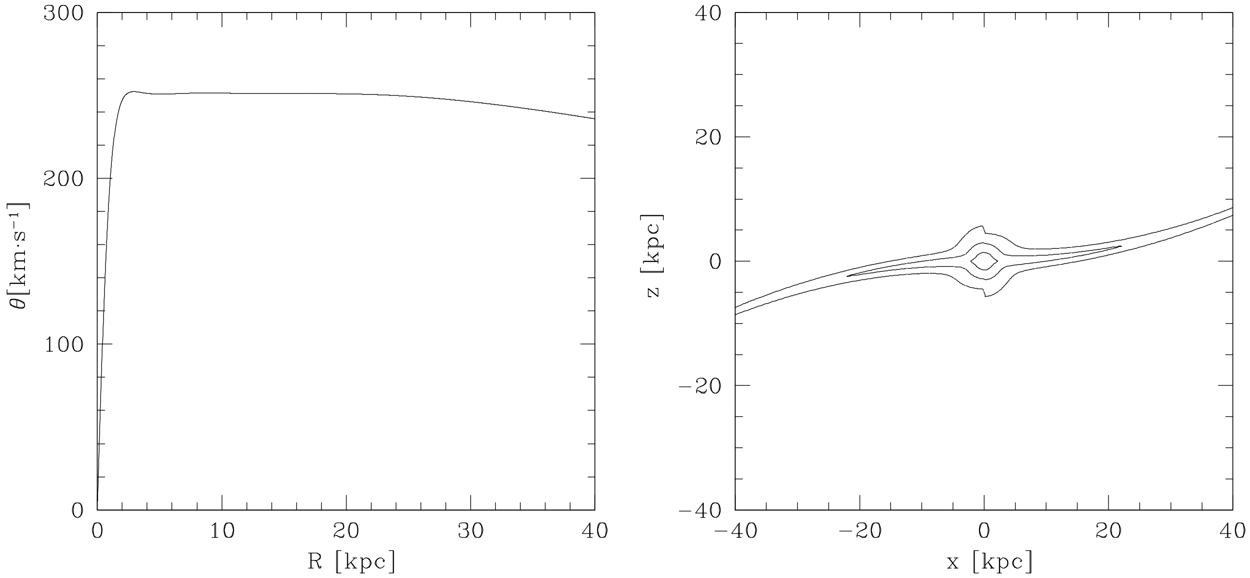
$$\Delta z = wR^2 \cos(\theta) \tag{2.1}$$

In cylindrical coordinates, the non axisymmetric potential Φ is given by :

$$\Phi_w(R,\theta,z) = -\sum_{i=1}^{3} \frac{GM_i}{\sqrt{R^2 + \left[a_i + \sqrt{(z - wR^2\cos(\theta))^2 + b_i^2}\right]^2}} \tag{2.2}$$

The parameters GM_i, the mass, a_i, the disk length scale, and b_i, the disk scale-height, are chosen to obtain a nearly constant velocity curve at large radii (40 kpc). The potential is also bi-symmetric with respect to the $y = 0$ plane ($\Phi(x, 0, z) = \Phi(-x, 0, -z)$). Fixing the total mass and G to a value of 1, the units are given in $5^{11}\mathrm{M}_\odot$, kpc and Myr. Figure shows the circular velocity and isodensity curves corresponding to the model.

In our model, we allow the possibility for a global rotation of the warped potential. This rotation is parametrised by the angular pattern speed Ω_p or, equivalently, by the corotating radius R_c, taking negative value for retrograde rotation.



(a) Circular velocity curve.

(b) Isodensity curves (from 10^{-3} to 10^{-5} $M_\odot \cdot$ kpc^{-3}, by steps of decades) in the $y = 0$ plane. $w = 0.005$ kpc^{-1}

2.2 Initial conditions for orbits

The orbit initial conditions are chosen with respect to the potential symmetries. The initial starting location is the y axis, equivalent to the line of nodes of the potential. In this study we consider only orbits symmetric with respect to the y-axis, therefore the initial velocity is perpendicular to the y axis, arbitrary pointing to negative values of x, see figure 2.1.

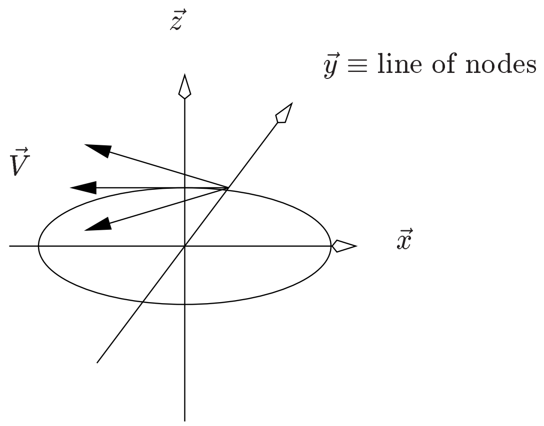


Fig. 2.1: Initial conditions for periodic orbits.

In this way, the free parameters in the initial conditions are the Jacobi constant ($\approx$ energy, H), the position along the y axis, and the velocity in the z direction (p_z).

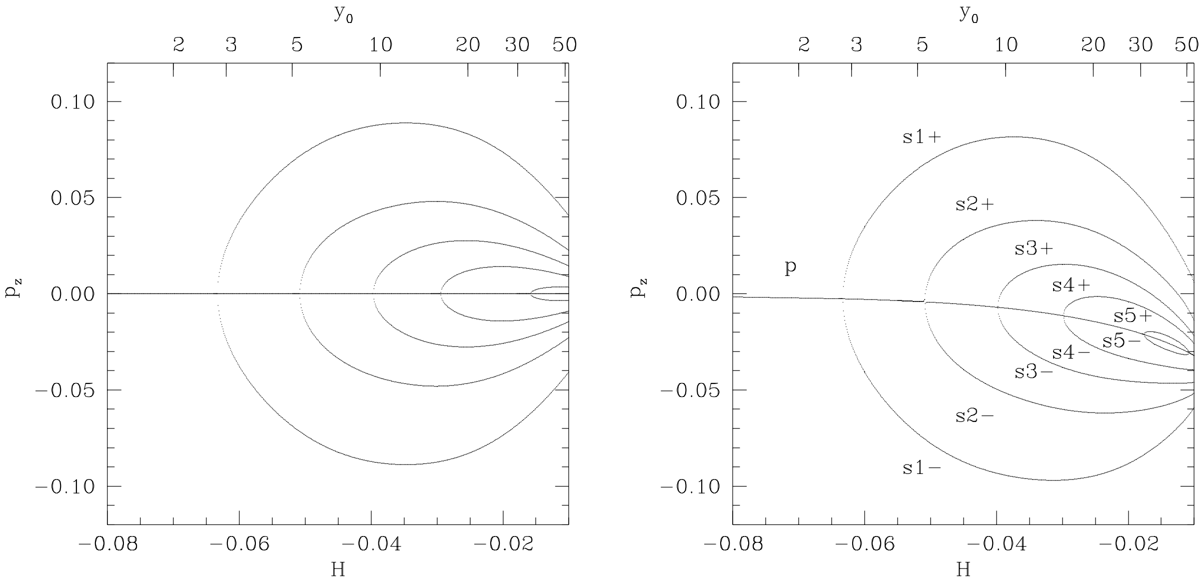
3 Numerical method

The periodic orbits are found by numerically finding the fixed points of the 4D Poincareé map at $y = 0$, $\dot{x} < 0$ generated by the equations of motion. The algorithm uses in particular the method proposed by Hénon (1982) giving the intersection between an orbit and a surface, and a least squares stabilized Newton-Raphson root finding procedure. A complete description is exposed in Pfenniger & Friedli (1991).

4 Results

4.1 Families of periodic orbits

We have found several families of periodic orbits well shown on the $H - p_z$ phase space (p_z velocity as a function of "energy" H. Figures compare orbit families of a plane disk ($w = 0$) to a slightly warped surface ($w = 0.005$) in the case of a non rotating potential ($\Omega_p = 0$). In figure (a), the straight line ($p_z = 0$) corresponds



(a) $w = 0$

(b) $w = 0.005$

Fig. 4.1: H-p_z phase space with and without deformation of the disc.

to the circular orbits family in the plane $z = 0$. Curves above and under this line correspond to orbits with initial p_z-velocity different from zero and also crossing the galactic plane several times.

In case of a deformation figure (b), family p, corresponding to the circular family, reduce the p_z velocity with respect to the initial radius y_0. The shape of periodic orbits are also quasi annular and follow to the higher density regions of the disc.

Family p is mostly stable, but at five bifurcations generates five new families of periodic orbits with same period that develop transverse to the warp.

4.1.1 Subfamilies

Subfamilies generated by bifurcations occur by pair ($s+$ and $s-$ "+" or "−" giving the sign of p_z) with respect to the p_z velocity. They are numered from 1 to 5 in order of the energy of bifurcation.

All families conserve the $y = 0$ plane symmetry, but only two (*s2* and *s4*) have the same symmetry as the potential.

They are all stable between $H = -0.08$ and $H = -0.01$ except families *s2*$-$ (unstable after $H = -0.046$) and family *s4*$-$ (unstable after $H = -0.025$). The three following figures show the shape of the families *s1*$+$, *s3*$+$ and *s4*$-$. Dashed lines represent the elevation of the warped disk as a function of the azimuthal angle θ.

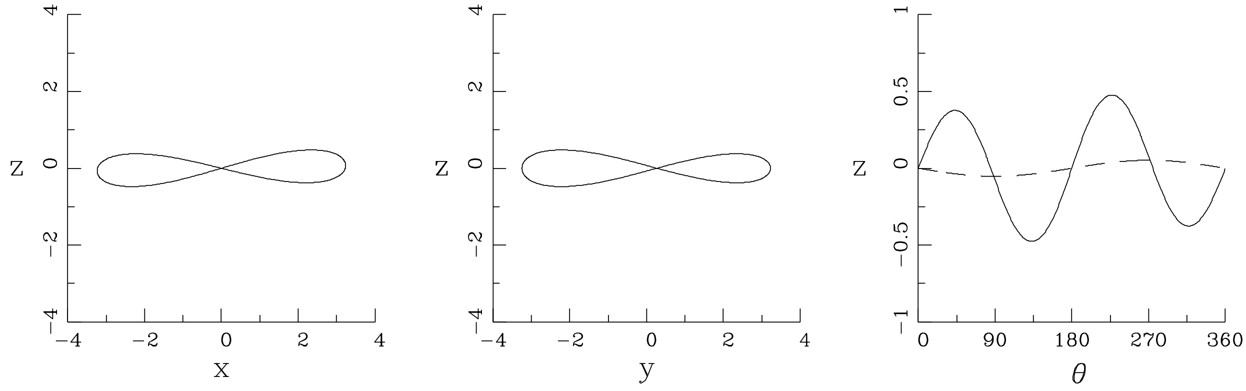


Fig. 4.2: family *s1+*

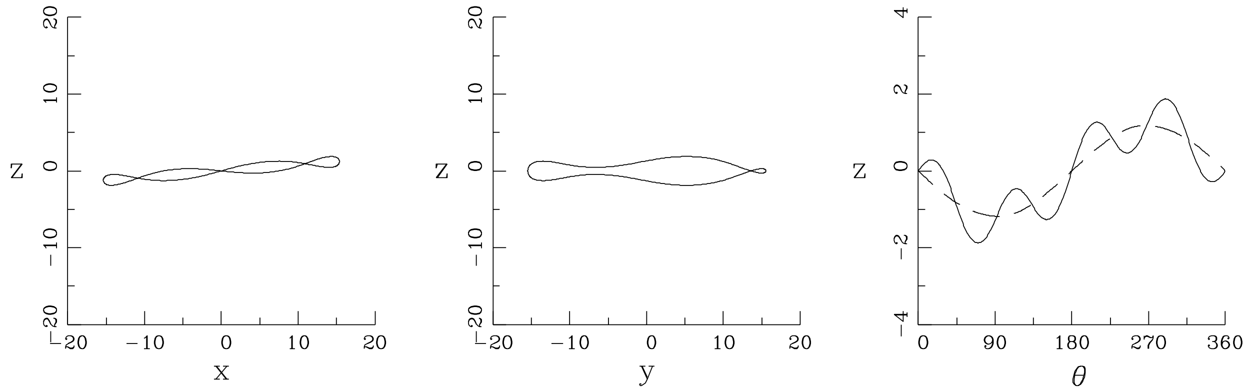


Fig. 4.3: family *s3+*

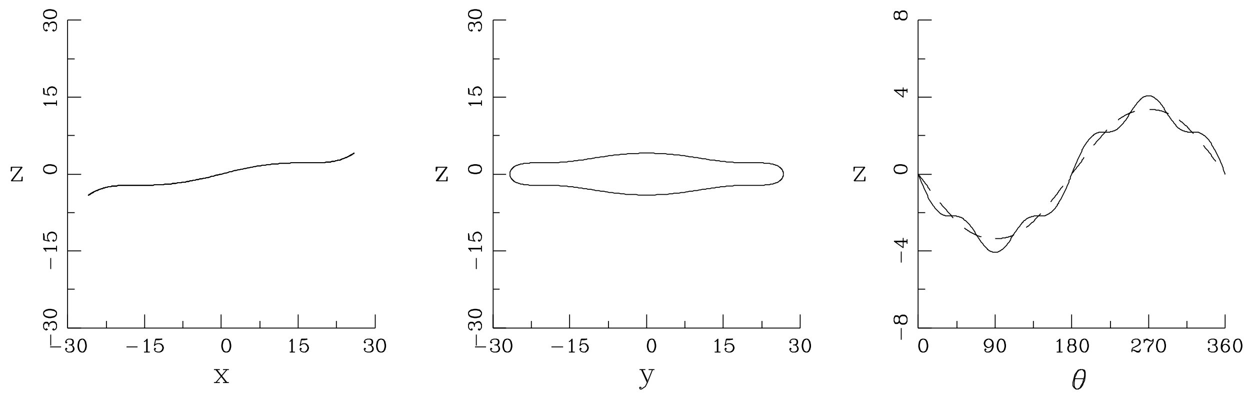


Fig. 4.4: family *s4−*

5 Consistency of the orbits with the mass density

If the warp is mostly self-gravitating and made of thin and distinct tube orbits, one can check the self-consistency constraint by noting that the spatial occupation of a periodic orbit is locally inversely proportional to its local speed (except at exceptional points). So density must be proportional to the inverse speed along the orbit ($\rho \sim 1/|v|$). Strictly this condition is only fulfilled by a structure entirely made of distinct exactly periodic orbits, such as a disk made of circular orbits. Nevertheless the check is useful in this problem because few hot orbits far from periodic round orbits are expected to exist.

We have tested the consistency of family p with respect to the energy including a global rotation. The results are given in figure 5.1.

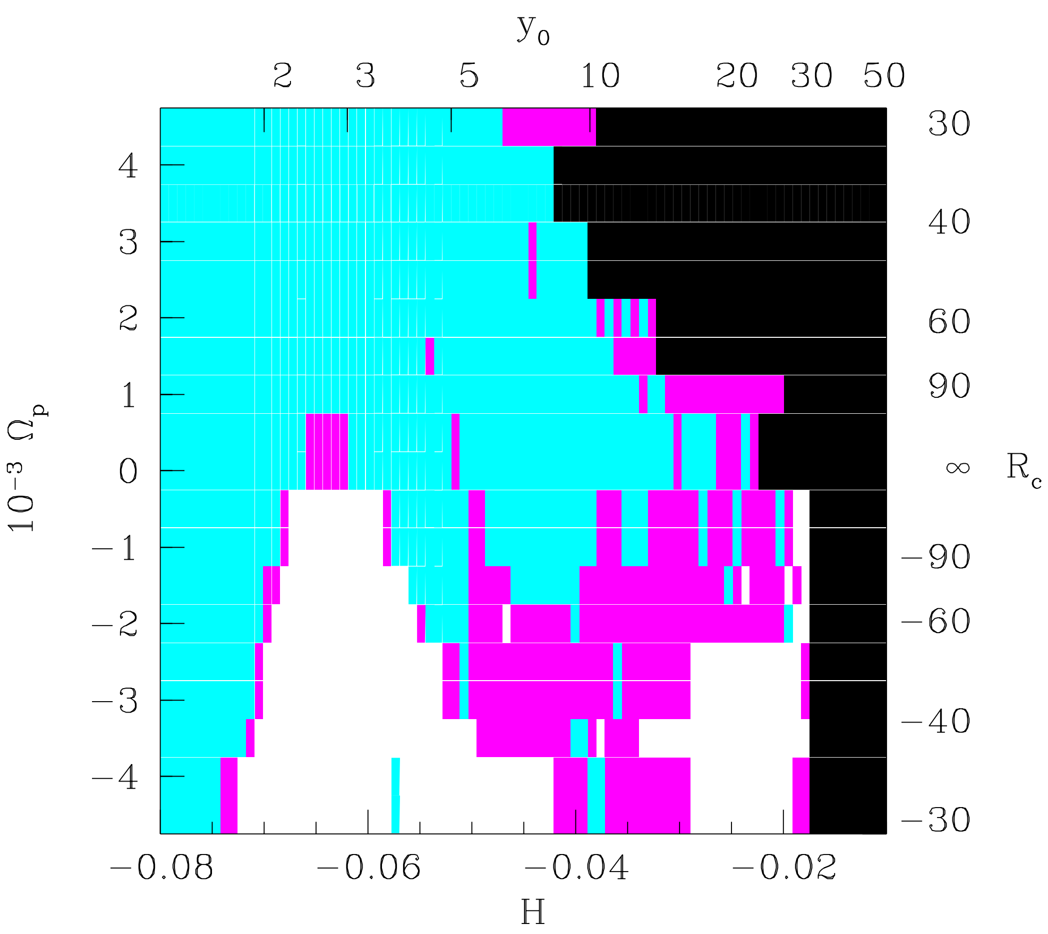


Fig. 5.1: Consistency of orbit occupation with the mass density. In white : the orbits reinforce the density ; in blue : the orbits destroy the density ; in pink : transition zones ; in black : missing data.

For a direct rotation (rotation in the sense of orbits), periodic orbits tend to destroy the density distribution. For retrograde rotation a consistent zone appear around 3 kpc. This zone grows with increasing rotation. A second zone appear around 20 kpc for rotations faster than $R_c = -60$ kpc. Finally, at $R_c = -30$, zones of consistence extend form 1 to 10 kpc and from 15 to 30 kpc. Note that other sub-families are clearly inconsistent with the density distribution, but they are also less relevant with the assumption of almost circular rotation.

6 Conclusions

- The quasi-annular orbit family survives to the warp perturbation and follows the warped galactic disk. This family corresponds to the circular family in an axisymmetric potential.
- Several transverse but thin resonances generating periodic orbit families exis. Most of them are stable.
- The influence of global rotation appears with the study of the consistence of the density distribution. A direct rotation of the warp pattern generates orbits destroying the density distribution, but on the contrary, orbits can locally reinforce the density in the presence of a retrograde rotation. This could helps holding external warped regions.

  A longer paper including an analysis of self-consistent N-body warps is in preparation.

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